Confinement of matter fields in compact (2+1)-dimensional QED theory of high-T_c superconductors

Guo-Zhu Liu
Department of Mathematics, University of Science and Technology of China, Hefei, Anhui, 230026, P.R. China

The confinement of matter fields is studied in the compact QED_3 theory of high-T_c superconductors. It is found that the monopole configurations do not affect the propagator of gauge potential a_μ. This then leads to the findings that chiral symmetry breaking and confinement take place simultaneously in the antiferromagnetic state and that neither monopole effect nor Anderson-Higgs mechanism can cause confinement in the d-wave superconducting state. The physical implications of these field theoretic results are also discussed.

PACS numbers: 74.20.Mn, 71.27.+a, 11.30.Qc, 14.80.Hv

The idea that high-T_c superconductor is some kind of quantum spin liquid[1] motivated much research effort in the past seventeen years. Based on slave-boson treatment of t-J model, it was found that the low-energy physics of the antiferromagnetic Mott insulator is captured by a theory of gapless Dirac fermions interacting with a U(1) gauge field[2,3]. Doping drives the Mott insulator to a d-wave superconductor, which can be described by a more general U(1) gauge theory including both Dirac fermions and scalar bosons[4]. The U(1) gauge field is not the usual electromagnetic field[2,3,4,5]. It originates from the strong correlation effect and is obtained in general by spontaneously breaking a larger SU(2) gauge symmetry. It has been proved[6] that cuprate superconductors at zero doping contain a local SU(2) gauge symmetry once slave-particle approach and the constraint of one particle per site are adopted. Later, Wen and Lee[7] constructed a SU(2) gauge structure away from half-filling. In the staggered flux phase, two components of the SU(2) gauge field become massive via Higgs mechanism and hence are neglected[7], leaving a massless U(1) gauge field. As claimed by Polyakov[8], this U(1) gauge field must be *compact* because the SU(2) group is defined on a compact sphere. The compact gauge structure also appears in theories of the Neel state and various spin liquid phases of the planar quantum Heisenberg antiferromagnets[9,10]. In particular, recently it has been used to build a critical theory of zero-temperature quantum phase transitions[10] that can not be described by the conventional Wilson-Ginzburg-Landau paradigm.

Polyakov[8] found that the (2+1)-dimensional compact quantum electrodynamics (QED_3) has monopole configurations around which gauge potential a_μ jumps $2n\pi$. The most remarkable effect of monopoles is that it leads to permanent confinement of static charges[11]. In order to understand realistic condensed matter systems, it is necessary to couple a_μ to fermions and scalar bosons. However, though the confinement in pure compact QED_3 is now widely accepted, the confinement of dynamical matter fields is far from clear. The confinement of massless fermions is of particular interests since they exist in the whole phase diagram of cuprate superconductors due to the d-wave gap symmetry. In our opinion, all previous efforts[2,12,13,14] towards the confinement problem of massless fermions are not satisfactory since they did not give a careful analysis of the relationship between chiral symmetry breaking (CSB), monopoles and confinement. Such an analysis is necessary not only in studying correlated electron systems but in understanding QCD, the theory of strong interactions.

In this paper we study the confinement of matter fields in the compact QED_3 theory of the high-T_c cuprate superconductors. We concentrate on the half-filled antiferromagnetic Mott insulator state and the d-wave superconducting state, two most interesting ground states in cuprate superconductors. Actually, the various strange behavior that can not be understood within conventional many-body theory are generally believed to arise from the competition between these two orders. We make a consistent treatment of confinement with CSB and monopole configurations considered on the same footing. The results are: 1) though the correlation function of magnetic field b_μ is affected by monopoles, the correlation function of gauge potential a_μ is not, indicating that compact QED_3 has the same perturbative (non-topological) structure with non-compact QED_3; 2) CSB and confinement that is caused by monopoles take place simultaneously in the half-filling antiferromagnetic state; 3) both CSB and single monopoles are suppressed in the d-wave superconducting state. We also argue that the Anderson-Higgs (AH) mechanism can not confine matter fields. The physical implications of these field theoretic results are also discussed in the context.

We first consider the low-energy effective theory of Heisenberg antiferromagnetism[2,3,4]. The Lagrangian is

$$\mathcal{L}_F = \sum_{\sigma=1}^{N} \overline{\psi}_\sigma \left(\partial_\mu - ia_\mu\right)\gamma_\mu\psi_\sigma + \frac{1}{4}F_{\mu\nu}^2. \qquad (1)$$

The fermion ψ_σ is a 4×1 spinor and the 4×4 γ_μ matrices obey the algebra $\{\gamma_\mu,\gamma_\nu\} = 2\delta_{\mu\nu}$. The Maxwell term $F_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu$ is kept here. It is convenient to write the action for the compact gauge field a_μ as

$$S_a \propto \sum_{x,\mu,\nu} \left(F_{\mu\nu} - 2\pi n_{\mu\nu}\right)\left(\frac{1}{2} + \pi(q)\right)\left(F_{\mu\nu} - 2\pi n_{\mu\nu}\right), \qquad (2)$$

where the $n_{\mu\nu}$ are integers. The term $1/2$ comes from the Maxwell term of a_μ and $\pi(q)$ is the vacuum polarization of fermions. If the fermions are massless, the vacuum polarization is $\pi(q) = N/8\,|q|$ to the one-loop approximation[4]. At large distances this term dominates and the term proportional to $1/2$ can be neglected. Then the action for a gas of monopoles of charge $q_a = \pm 1$ is

$$S_{mono} = \frac{\pi^2 N}{4} \sum_{a,b} q_a q_b V(\mathbf{x_a} - \mathbf{x_b}). \qquad (3)$$

The potential $V(\mathbf{x})$ between monopoles is

$$V(\mathbf{x}) = \int \frac{d^3k}{(2\pi)^3} \frac{e^{ikx}}{k^3} \sim \ln|\mathbf{x}|. \qquad (4)$$

Since the monopoles interact with a logarithmic potential, a Kosterlitz-Thouless (KT) transition would take place at some critical flavor N_{cf} below which single monopoles are proliferated. However, there is a controversy on N_{cf}. Some authors found that $N_{cf} = 24$[2,13] while others found that $N_{cf} = 0.9$[12]. We should emphasize that, even if single monopoles are important, we can not immediately draw the conclusion that massless fermions are confined. The reason is that Wilson's area law[11] was proposed to describe confinement of pure gauge field and static charge sources. It loses its meaning when the gauge field couples to dynamical massless fermions. When the sources separate, it becomes more favorable to create a pair of fermions which then screens the gauge force[15]. However, if the massless fermions acquire a finite mass via the CSB mechanism they then can be considered as static sources. Indeed, when the fermions become massive, creating a fermion-antifermion pair out of the vacuum would cost a large amount of energy.

CSB can be studied by the standard Dyson-Schwinger (DS) equation approach. The propagator of massless fermions is $S^{-1}(p) = i\gamma \cdot p$. Interaction with gauge field renormalizes it to $S^{-1}(p) = i\gamma \cdot pA\left(p^2\right) + \Sigma\left(p^2\right)$ with $A(p^2)$ the wave function renormalization and $\Sigma(p^2)$ the fermion self-energy. The self-energy $\Sigma(p^2)$ satisfies a set of self consistent DS equations, which to the lowest order in $1/N$ expansion has the simple form

$$\Sigma(p^2) = \int \frac{d^3k}{(2\pi)^3} \frac{\gamma^\mu D_{\mu\nu}(p-k)\Sigma(k^2)\gamma^\nu}{k^2 + \Sigma^2(k^2)}, \qquad (5)$$

where a bare vertex is adopted[16]. If this equation has only vanishing solutions, the gauge field is an irrelevant perturbation and fermions remain massless. If $\Sigma(p^2)$ develops a nontrivial solution, the massless fermions acquire a finite mass which breaks the chiral symmetry of Lagrangian (1). For non-compact gauge field, the propagator in the Landau gauge is

$$D_{\mu\nu}(q) = \frac{1}{q^2}\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right). \qquad (6)$$

It was found[16] that CSB can only take place for $N < N_c = 32/\pi^2$. The CSB in noncompact QED$_3$ has been used to understand many properties of cuprate superconductors[4,17].

To investigate the DS equation in compact QED$_3$, we should first know the effect of monopoles on the propagator of a_μ. According to the arguments of Polyakov, the magnetic field correlation function[8] is

$$\langle b_\mu b_\nu \rangle = \langle b_\mu b_\nu \rangle_0 + \langle b_\mu b_\nu \rangle_m, \qquad (7)$$

where $\langle b_\mu b_\nu \rangle_0$ is the propagator without monopoles and $\langle b_\mu b_\nu \rangle_m$ is the contribution of monopoles. The density operator of monopoles is $\rho(x) = \sum_a q_a \delta(x - x_a)$ which is related to the magnetic field as $b_\mu(x) = \frac{1}{2}\int d^3y \frac{(x-y)_\mu}{|x-y|^3}\rho(y)$ or $b_\mu(q) = \frac{q_\mu}{q^2}\rho(q)$ in the momentum space. The singular contribution to the magnetic filed correlation function is

$$\langle b_\mu b_\nu \rangle_m = \frac{q_\mu q_\nu}{q^4}\langle \rho(q)\rho(-q)\rangle = \frac{q_\mu q_\nu}{q^4}\frac{M^2 q^2}{q^2 + M^2}. \qquad (8)$$

Then we have

$$\langle b_\mu b_\nu \rangle = \delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} + \frac{q_\mu q_\nu}{q^2}\frac{M^2}{q^2+M^2} = \delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2+M^2}. \qquad (9)$$

The appearance of a pole at $-M^2$ was interpreted by Polyakov[8] as the evidence of a finite mass gap for compact gauge field.

In quantum gauge field theories, it is the gauge potential a_μ that couples directly to dynamical matter fields. Therefore, to study CSB we should calculate the propagator of a_μ. The magnetic field b_μ is related to a_μ as $b_\mu = \epsilon_{\mu\nu\lambda} q_\nu a_\lambda$. Then the correlation function of a_μ is

$$D_{\mu\nu}(q) = \langle a_\mu a_\nu \rangle = \epsilon_{\mu ij}\epsilon_{\nu kl}\frac{q_i q_k}{q^4}\langle b_j b_l \rangle. \qquad (10)$$

Using the fact that

$$\epsilon_{\mu ij}\epsilon_{\nu kl}\frac{q_i q_k}{q^4}\langle b_j b_l \rangle_m = \epsilon_{\mu ij}\epsilon_{\nu kl}\frac{q_i q_k q_j q_l}{q^6}\frac{M^2}{q^2+M^2} = 0. \qquad (11)$$

Then we get the propagator

$$D_{\mu\nu}(q) = \epsilon_{\mu ij}\epsilon_{\nu kl}\frac{q_i q_k}{q^4}\langle b_j b_l \rangle_0 = \frac{1}{q^2}\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right). \qquad (12)$$

It is clear that the monopole configurations do not affect the propagator of a_μ. Although magnetic field b_μ is the quantity that can be detected directly by experiments, a_μ is the physical quantity that couples directly to matter fields. Therefore, the monopole configurations do not affect the interaction of a_μ with matter fields, at least within the perturbation theory. This might not be too surprising if we note the fact that a_μ always interacts *locally* with matter fields but the monopoles reflect the nontrivial topology of the gauge field configuration which is certainly a *global* property.

The results about CSB obtained in non-compact QED$_3$[18] also applies to compact QED$_3$. If the flavor

of massless fermions $N < N_c$, CSB takes place, while if $N > N_c$ the fermions remain massless and the chiral symmetry is respected. Although there is a little debate on the value of N_c, most analytic and numerical calculations indicated that it is about $3.3 \sim 4$. For $N < N_c$, the massless fermions becomes massive and hence its contribution to the vacuum polarization is

$$\pi(q) = \frac{N}{4\pi}\left(\frac{2m}{q^2} + \frac{q^2 - 4m^2}{q^2|q|}\arcsin\left(\frac{q^2}{q^2+4m^2}\right)^{1/2}\right). \quad (13)$$

Here we adopt a constant fermion mass m for simplicity. We only care about the behavior at very low momentum limit since confinement is essentially a phenomenon of large distances. It is easy to see that $\pi(q) \to N/8\pi m$ in the $q \to 0$ limit. Obviously, the only effect of massive fermions on the action of monopoles is a renormalization of the gauge coupling constant. Consequently, the monopoles are in the Coulomb gas phase, just like in the pure compact gauge theory. Since fermions are massive there are undoubtedly no fermion zero modes which, if exist, would suppress the monopole configurations[12]. The massive fermions can be approximately considered as static charges. Then Wilson's confinement criteria can be used safely. Confinement was found[8] unambiguously after calculating the Wilson loop $F[C] =< e^{i\oint a_\mu dx_\mu} >$. Therefore, CSB and confinement take place simultaneously for $N < N_c$[19]. For $N > N_c$, a careful analysis of KT transition and deeper insights on the criteria of confinement for massless fermions are needed[20].

For cuprate superconductors, the physical flavor is $N = 2 < N_c$, corresponding to the two components of spin 1/2. Then CSB and confinement both occur at half-filling and prevent the appearance of mobile fermions at low temperatures. This is consistent with the fact that the low temperature thermal conductivity vanishes at very low doping concentrations[21]. CSB generates a finite gap for the gapless nodal fermions. This accounts for the finite nodal gap observed by angle-resolved photoemission spectroscopy (ARPES) measurements in lightly doped cuprates[22]. Moreover, when CSB happens, a long-range antiferromagnetic order is formed, corresponding to the well-known Néel order of undoped cuprates[4,17].

We next would like to consider confinement of matter fields in the d-wave superconducting state at finite doping concentration. To describe the d-wave superconductor, we should couple both massless Dirac fermions and holons ϕ to the U(1) gauge field a_μ. The scalar field ϕ develops a nonzero vacuum expectation ($\langle\phi\rangle \neq 0$) and the gauge boson a_μ acquires a finite mass ξ via AH mechanism. The gauge boson mass ξ suppresses CSB completely[23], hence the low-energy excitations are gapless nodal fermions. (Note that there should not be a Yukawa coupling term $\phi\overline{\psi}\psi$ between massless fermions and scalar field ϕ. If such a term were present, then the nonzero $\langle\phi\rangle$ would generate a finite mass for the massless fermions, in disagreement with experiments).

Using a simple but compelling argument, we can show that single monopoles can not exist in a superconductor. When a single monopole is placed in the interior of a superconductor its line of magnetic flux must have somewhere to go. However, according to the Meissner effect, a superconductor always repels the magnetic field. Thus the magnetic flux emitting from a monopole must end at an anti-monopole. In other words, the monopoles must appear in the form of bound pairs and all the magnetic flux is trapped into a thin tube. To see this more explicitly, we can calculate the potential between two monopoles in a superconductor. The propagator of massive gauge boson is

$$D_{\mu\nu}(q) = \frac{1}{q^2\left[1 + \pi(q^2) + \xi^2 q^{-2}\right]}\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right). \quad (14)$$

The gauge boson mass term $\xi^2 q^{-2}$ dominates the low momentum behavior, no matter whether the fermions are massless or not. Using the same calculations that lead to (4), we found a linear potential

$$V(\mathbf{x}) = \int \frac{d^3k}{(2\pi)^3}\frac{e^{ikx}}{k^4} \sim |\mathbf{x}| \quad (15)$$

between monopoles. Therefore, single monopoles can not exist and there is a string between a monopole and an anti-monopole. The superconductor can be understood by the picture that condensation of charged particles gives rise to confinement of magnetic monopoles. If we interchange the roles of electricity and magnetism, then we get a dual picture that condensation of magnetic monopoles causes confinement of charged particles, which describes the half-filled antiferromagnetic state. Thus an "electromagnetic" duality exist between the Heisenberg antiferromagnet and the superconductor, which might help us to understand the physics of cuprate superconductors. This kind of duality also underlies the most exciting attempts[24] made recently towards a final understanding of quark confinement in QCD.

The spin-charge separation and recombination have been studied extensively[4,7,25,26]. It is generally expected that spinons and holons are bound together to form real electrons in the d-wave superconductor. Two possible ways have been proposed[26] to realize the confinement: AH mechanism and monopole effect. Since single monopoles do not exist in a superconductor, it seems natural that it is the AH mechanism that causes confinement. However, we believe that this is not the case. Remember that Higgs mechanism (the non-Abelian generalization of Abelian AH mechanism) appears in the standard model of electro-weak interaction[27]. Although the intermediate gauge bosons acquire finite mass gap, the fermions and the gauge bosons are certainly not confined. Confinement via AH mechanism requires that the gauge coupling must be very strong at the $q \to 0$ limit. For QED_3, we can define a dimensionless running gauge coupling $\overline{\alpha}(q)$ as[18]

$$\overline{\alpha}(q) = \frac{e^2 q}{q^2 + \xi^2 + (e^2N/8)q} = \frac{8}{N}\frac{\alpha q}{q^2 + \xi^2 + \alpha q}. \quad (16)$$

The running coupling constant $\overline{\alpha}(q)$ vanishes at both $q \rightarrow \infty$ and $q \rightarrow 0$ limits. Since the gauge coupling is weakened by the gauge boson mass generated via AH mechanism, the matter fields should not be confined. We can make a comparison between the coupling strengths that are needed to cause CSB and confinement. Suppose that CSB takes place, then the potential between a fermion and an anti-fermion has a logarithmic form[28], $V(\mathbf{x}) \sim \frac{\ln|\mathbf{x}|}{1+\pi(0)} \sim \ln|\mathbf{x}|$, with $\pi(0)$ the vacuum polarization of massive fermions at zero momentum. But in general confinement requires a linear potential $V(\mathbf{x}) \sim |\mathbf{x}|$ between two particles. The attractive force that is needed to cause confinement should be much stronger than that needed to cause CSB. In general, when the gauge boson acquires a finite mass via AH mechanism, its coupling is not strong enough to cause CSB[23]. Thus it certainly can not cause confinement.

We now see that both monopole effect and AH mechanism can not be the confining mechanism that leads to spin-charge recombination. This leaves us with two possibilities: confinement is caused by a new unknown mechanism or it does not occur in the superconducting state. The later possibility is not impossible. At present, almost all ARPES experiments supporting the existence of well-defined quasiparticle peaks in superconducting state have been performed in the $(\pm\pi, 0)$ directions[29]. Recent ARPES experiments in the $(\pm\pi/2, \pm\pi/2)$ directions revealed a much shorter quasiparticle lifetime than that was predicted by BCS-like theory[29,30]. Other evidence for the existence of well-defined quasiparticles comes from the finite thermal conductivity at low temperatures observed by heat transport measurements[31]. However, the heat transport behavior could also be described by the spinons and spin-charge recombination is not required. To tell which possibility actually works needs further investigations.

It was showed[15] that the Higgs and confining phases are smoothly connected when the Higgs fields transform like the fundamental representation of the gauge group. While this result applies to other lattice gauge theories, it does not apply to the compact QED_3. In the Higgs phase, a true gauge boson mass gap is generated by vacuum degeneracy, while in the confining phase there is no vacuum degeneracy and the monopoles only affect the correlation function of b_μ, leaving that of a_μ unchanged.

The results in this paper can understand some physics of high-T_c superconductors from a field theoretic point of view and are also helpful in studying confinement of more complicated gauge theories such as QCD.

This work is supported by the NSF of China No. 10404024 and the China Postdoctoral Science Foundation.

[1] P. W. Anderson, Sicnece **235**, 1196 (1987).
[2] L. B. Ioffe and A. I. Larkin, Phys. Rev. B **39**, 8988 (1989).
[3] J. B. Marston and I. Affleck, Phys. Rev. B **39**, 11538 (1989).
[4] D. H. Kim and P. A. Lee, Ann. Phys. (N.Y.) **272**, 130 (1999).
[5] G. Baskaran and P. W. Anderson, Phys. Rev. B **37**, R580 (1988).
[6] I. Affleck, Z. Zou, T. Hsu, and P. W. Anderson, Phys. Rev. B **38**, 745 (1988); E. Dagotto, E. Fradkin, and A. Moreo, Phys. Rev. B **38**, 2926 (1988).
[7] X.-G. Wen and P. A. Lee, Phys. Rev. Lett. **76**, 503 (1996); P. A. Lee, N. Nagaosa, T. K. Ng, and X.-G. Wen, Phys. Rev. B **57**, 6003 (1998).
[8] A. M. Polyakov, Nucl. Phys. B **120**, 429 (1977).
[9] S. Sachdev and K. Park, Ann. Phys. (N. Y.) **298**, 58 (2002).
[10] T. Senthil, L. Balents, S. Sachdev, A. Vishwanath, and M. P. A. Fisher, Phys. Rev. B **70**, 144407 (2004).
[11] K. G. Wilson, Phys. Rev. D **10**, 2445 (1974).
[12] J. B. Marston, Phys. Rev. Lett. **64**, 1166 (1990).
[13] H. Kleinert, F. S. Nogueira, and A. Sudbo, Phys. Rev. Lett. **88**, 232001 (2002).
[14] I. F. Herbut and B. H. Seradjeh, Phys. Rev. Lett. **91**, 171601 (2003).
[15] E. Fradkin and S. H. Shenker, Phys. Rev. D **19**, 3682 (1979).
[16] T. Appelquist, D. Nash, and L. C. R. Wijewardhana, Phys. Rev. Lett. **60**, 2575 (1988).
[17] G.-Z. Liu and G. Cheng, Phys. Rev. B **66**, 100505(R) (2002); Z. Tešanović, O. Vafek, and M. Franz, Phys. Rev. B **65**, 180511(R) (2002).
[18] For a review on CSB in QED_3, see T. Appelquist and L. C. R. Wijewardhana, hep-ph/0403250.
[19] The kinetic term $F_{\mu\nu}^2$ should be taken away when we consider the effective theory of cuprate superconductors[2,3,4]. However, it is easy to see that the conclusion does not change.
[20] It is claimed that the monopoles can be irrelevant in some U(1) spin liquids based on an SU(N) generalization of the $S = 1/2$ square lattice Heisenberg antiferromagnets. See M. Hermele, T. Senthil, M. P. A. Fisher, P. A. Lee, N. Nagaosa, and X.-G. Wen, Phys. Rev. B **70**, 214437 (2004).
[21] M. Sutherland, D. G. Hawthorn, R. W. Hill, F. Ronning, S. Wakimoto, H. Zhang, C. Proust, E. Boaknin, C. Lupien, L. Taillefer, R. Liang, D. A. Bonn, W. N. Hardy, R. Gagnon, N. E. Hussey, T. Kimura, M. Nohara, and H. Takagi, Phys. Rev. B **67**, 174520 (2003).
[22] K. M. Shen, T. Yoshida, D. H. Lu, F. Ronning, N. P. Armitage, W. S. Lee, X. J. Zhou, A. Damascelli, D. L. Feng, N. J. C. Ingle, H. Eisaki, Y. Kohsaka, H. Takagi, T. Kakeshita, S. Uchida, P. K. Mang, M. Greven, Y. Onose, Y. Taguchi, Y. Tokura, S. Komiya, Y. Ando, M. Azuma, M. Takano, A. Fujimori, and Z.-X. Shen, Phys. Rev. B **69**, 054503 (2004).
[23] G.-Z. Liu and G. Cheng, Phys. Rev. D **67**, 065010 (2003).
[24] G. 't Hooft, hep-th/0010225; N. Seiberg and E. Witten, Nucl. Phys. B **426**, 19 (1994).
[25] C. Mudry and E. Fradkin, Phys. Rev. B **49**, 5200 (1994).
[26] W. Rantner and X.-G. Wen, Phys. Rev. Lett. **86**, 3871 (2001).
[27] S. Weinberg, Phys. Rev. Lett. **19**, 1264 (1967).

[28] P. Maris, Phys. Rev. D **52**, 6087 (1995).
[29] J. Orenstein and A. J. Millis, Science **288**, 468 (2000).
[30] T. Valla, A. V. Fedorov, P. D. Johnson, B. O. Wells, S. L. Hulbert, Q. Li, G. D. Gu, and N. Koshizuka, Science **285**, 2110 (1999); J. Corson, J. Orenstein, S. Oh, J. O'Donnell, and J. N. Eckstein, Phys. Rev. Lett. **85**, 2569 (2000).
[31] L. Taillefer, B. Lussier, R. Gagnon, K. Behnia, and H. Aubin, Phys. Rev. Lett. **79**, 483 (1997).